GACE® Mathematics Assessment
Test at a Glance
Updated February 2022

See the GACE® Mathematics Assessment Study Companion for practice questions and preparation resources.

Assessment Name	Mathematics
Grade Level	6–12
Test Code	Test I: 022 Test II: 023 Combined Test I and Test II: 522
Testing Time	Test I: 2 hours 15 minutes Test II: 2 hours 15 minutes Combined Test I and Test II: 4.5 hours
Test Duration	Test I: 2.5 hours Test II: 2.5 hours Combined Test I and Test II: 5 hours
Test Format	Computer delivered
Number of Selected-response Questions	Test I: 50 Test II: 50 Combined Test I and Test II: 100
Question Format	The test consists of a variety of short-answer questions such as selected-response questions, where you select one answer choice or multiple answer choices (depending on what the question asks for), questions where you enter your answer in a text box, and other types of questions. You can review the possible question types in the *Guide to Taking a GACE Computer-delivered Test*.
Number of Constructed-response Questions	Test I: 0 Test II: 0 Combined Test I and Test II: 0

About this Assessment

The GACE Mathematics assessment is designed to measure the professional knowledge of prospective teachers of 6–12 mathematics in the state of Georgia.

This assessment includes two tests. You may take either test individually or the full assessment in a single session. The testing time is the amount of time you will have to answer the questions on the test. Test duration includes time for tutorials and directional screens that may be included in the test.

The questions in this assessment assess both basic knowledge across content areas and the ability to apply principles.

The total number of questions that are scored is typically smaller than the total number of questions on the test. Most tests that contain selected-response questions also include embedded pretest questions, which are not used in calculating your score. By including pretest questions in the assessment, ETS is able to analyze actual test-taker performance on proposed new questions and determine whether they should be included in future versions of the test.

Content Specifications

Each test in this assessment is organized into content **subareas**. Each subarea is further defined by a set of **objectives** and their **knowledge statements**.

- The objectives broadly define what an entry-level educator in this field in Georgia public schools should know and be able to do.
- The knowledge statements describe in greater detail the knowledge and skills eligible for testing.
- Some tests also include content material at the evidence level. This content serves as descriptors of what each knowledge statement encompasses.

See a breakdown of the subareas and objectives for the tests in this assessment on the following pages.

Test I Subareas

Subarea	Approx. Percentage of Test
I. Number and Quantity	30%
II. Algebra	40%
III. Discrete Mathematics and Calculus	30%

Test I Objectives

Subarea I: Number and Quantity

Objective 1: Understands and applies knowledge of the real number system and vector and matrix quantities

The beginning Mathematics teacher:

A. Understands the properties of exponents

- Performs operations involving exponents, including negative and rational exponents
- Demonstrates an understanding of the properties of exponential expressions
- Uses the properties of exponents to rewrite expressions that have radicals or rational exponents

B. Understands the properties of rational and irrational numbers and the interactions between those sets of numbers

- Recognizes that the sum or product of two rational numbers is rational
- Recognizes that the sum of a rational number and an irrational number is irrational
- Recognizes that the product of a nonzero rational number and an irrational number is irrational
- Recognizes that the sum or product of two irrational numbers can be rational or irrational

C. Is familiar with the representation and modeling of vector quantities and how operations on vectors are performed

- Represents vector quantities by directed line segments and uses appropriate symbols for vectors and their magnitudes
- Finds the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point

- Solves problems involving velocity and other quantities that can be represented by vectors

- Adds vectors end-to-end, component-wise, and by the parallelogram rule

- Given two vectors in magnitude and direction form, determines the magnitude and direction of their sum

D. Understands how to perform operations on matrices and how to use matrices in applications

- Uses matrices to represent and manipulate data

- Multiplies matrices by scalars to produce new matrices

- Adds, subtracts, and multiplies matrices of appropriate dimensions

- Understands that matrix multiplication for square matrices is not a commutative operation but still satisfies the associative and distributive properties

- Understands the role played by zero and identity matrices in matrix addition and multiplication

- Understands that the determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse

E. Understands how to solve problems involving ratios, proportions, averages, percents, and metric and traditional unit conversions

- Applies the concept of a ratio and uses ratio language and notation to describe a relationship between two quantities and solve problems

- Uses ratio reasoning to convert rates

- Solves problems involving scale factors

- Recognizes and represents proportional and inversely proportional relationships between two quantities

- Uses proportional relationships to solve multistep ratio, average, and percent problems

- Solves measurement and estimation problems involving time, length, temperature, volume, and mass in both the U.S. customary system and the metric system, where appropriate

- Converts units within the metric and customary systems

F. Understands various ways to represent, compare, estimate, and perform calculations on very large and very small numbers; e.g., scientific notation, orders of magnitude

- Represents and compares very large and very small numbers
- Uses orders of magnitude to estimate very large and very small numbers
- Performs calculations on numbers in scientific notation

Objective 2: Understands and applies knowledge of quantities and the complex number system

The beginning Mathematics teacher:

A. Understands how to solve problems by reasoning quantitatively; e.g., dimensional analysis, reasonableness of solutions

- Uses units as a way to understand problems and to guide the solution of multistep problems
- Chooses and interprets units consistently in formulas
- Chooses and interprets the scale and the origin in graphs and data displays
- Recognizes the reasonableness of results within the context of a given problem

B. Understands the structure of the natural, integer, rational, real, and complex number systems and how the basic operations ($+$, $-$, $\times$, and $\div$) on numbers in these systems are performed

- Solves problems using addition, subtraction, multiplication, and division of rational, irrational, and complex numbers
- Applies the order of operations
- Given operations on a number system, determines whether the properties (e.g., commutative, associative, distributive) hold
- Compares, classifies, and orders real numbers
- Demonstrates an understanding of the properties of counting numbers; e.g., prime, composite, prime factorization, even, odd, factors, multiples

C. Knows how complex numbers and operations on them are represented in the complex plane

- Represents complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers)
- Explains why the rectangular and polar forms of a given complex number represent the same number

- Represents addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane, and uses properties of the representation for computation
- Calculates the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints

D. Understands how to work with complex numbers when solving polynomial equations and rewriting polynomial expressions

- Solves quadratic equations with real coefficients that have complex solutions
- Extends polynomial identities to the complex numbers; e.g., $x^2 + y^2 = (x + yi)(x - yi)$

E. Knows how to analyze both precision and accuracy in measurement situations

- Chooses a level of accuracy appropriate to limitations on measurement when reporting quantities
- Calculates or estimates absolute and relative error in the numerical answer to a problem

Subarea II: Algebra

Objective 1: Sees structure in expressions and understands arithmetic with polynomials and rational expressions

The beginning Mathematics teacher:

A. Understands how to write algebraic expressions in equivalent forms

- Uses the structure of an expression to identify ways to rewrite it
- Understands how to rewrite quadratic expressions for specific purposes; e.g., factoring/finding zeros, completing the square/finding maxima or minima
- Uses the properties of exponents to rewrite expressions for exponential functions

B. Understands how to perform arithmetic operations on polynomials

- Adds, subtracts, multiplies, and divides polynomials

C. Understands the relationship between zeros of polynomial functions (including their graphical representation) and factors of the related polynomial expressions

- Knows and applies the remainder theorem: for a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $x - a$ is a factor of $p(x)$

- Uses factorization to identify zeros of polynomials
- Uses zeros of a polynomial to construct a rough graph of the function defined by the polynomial

D. Understands how to use the binomial theorem to solve problems

- Applies the binomial theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n

E. Understands how to rewrite rational expressions and perform arithmetic operations on rational expressions

- Rewrites simple rational expressions in different forms
- Understands that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression
- Adds, subtracts, multiplies, and divides rational expressions

F. Understands the properties of number systems under various operations

- Given operations on algebraic expressions, determines whether properties (e.g., commutative, associative, distributive) hold
- Performs calculations using newly defined functions

Objective 2: Understands how to create equations and how to reason with equations and inequalities

The beginning Mathematics teacher:

A. Understands how to create equations and inequalities that describe relationships

- Creates equations and inequalities in one variable and uses them to solve problems and graph solutions on the number line
- Creates equations and inequalities to represent relationships between quantities, solves problems, and graphs them on the coordinate plane with labels and scales
- Represents constraints by equations, inequalities, or systems of equations and/or inequalities, and interprets solutions as viable or nonviable options in a modeling context
- Rearranges formulas to highlight a quantity of interest; e.g., solve $d = rt$ for t

B. Understands how to justify the reasoning process used to solve equations, including analysis of potential extraneous solutions

- States each step in solving a simple equation
- Solves simple rational and radical equations in one variable, incorporating analysis of possible extraneous solutions

C. Understands how varied techniques (e.g., graphical, algebraic) are used to solve equations and inequalities

- Solves linear equations and inequalities, including equations with coefficients represented by letters

- Uses the method of completing the square to transform any quadratic equation in x into the equivalent form $(x - p)^2 = q$

- Solves equations using a variety of methods (e.g., using graphs, using the quadratic formula, factoring)

- Uses different methods (e.g., discriminant analysis, graphical analysis) to determine the nature of the solutions of a quadratic equation

D. Understands how varied techniques (e.g., graphical, algebraic, matrix) are used to solve systems of equations and inequalities

- Explains why, when solving a system of two equations using the elimination method, replacing one or both equations with a scalar multiple produces a system with the same solutions as the solutions of the original system

- Solves a system consisting of two linear equations in two variables algebraically and graphically

- Solves a system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically

- Represents a system of linear equations as a single matrix equation

- Finds the inverse of a matrix if it exists and uses it to solve systems of linear equations

- Explains why the x-coordinates of the intersection points of the graphs of $y = f(x)$ and $y = g(x)$ are the solutions of $f(x) = g(x)$

- Finds the solutions of $f(x) = g(x)$ approximately (e.g., uses technology to graph the functions, makes tables of values, finds successive approximations); includes cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, or logarithmic functions

- Graphs the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graphs the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes

E. Understands the concept of rate of change of nonlinear functions

- Calculates and interprets the average rate of change of a function presented symbolically, numerically, or graphically over a specified interval

F. Understands the concepts of intercept(s) of a line and slope as a rate of change

- Calculates and interprets the intercepts of a line
- Calculates and interprets the slope of a line presented symbolically, numerically, or graphically
- Estimates the rate of change of a linear function from a graph

G. Understands how to find the zero(s) of functions

- Uses a variety of techniques to find and analyze the zero(s) (real and complex) of functions

Subarea III: Discrete Mathematics and Calculus

Objective 1: Understands and applies knowledge of discrete mathematics

The beginning Mathematics teacher:

A. Understands sequences; e.g., arithmetic, recursively defined, geometric

- Writes arithmetic and geometric sequences both recursively and with an explicit formula, uses them to model situations, and translates between the two forms
- Evaluates, extends, or algebraically represents rules that involve number patterns
- Explores patterns in order to make conjectures, predictions, or generalizations

B. Understands the differences between discrete and continuous representations (e.g., data, functions) and how each can be used to model various phenomena

- Understands the differences between discrete and continuous representations; e.g., data, functions
- Understands how discrete and continuous representations can be used to model various phenomena

C. Knows how to model and solve problems using vertex-edge graphs, trees, and networks

- Constructs, uses, and interprets simple diagrams to solve problems
- Solves linear programming problems

D. Understands basic terminology and symbols of logic

- Understands the basic terminology of logic
- Uses logic to evaluate the truth of statements
- Uses logic to evaluate the equivalence of statements; e.g., statement and contrapositive

- Identifies basic properties of quantifiers; e.g., for all, there exists

- Negates statements involving quantifiers; e.g., for all, there exists

E. Understands how to use counting techniques such as the multiplication principle, permutations, and combinations

- Uses counting techniques to solve problems

F. Understands basic set theory; e.g., unions, differences, and Venn diagrams

- Solves problems using basic set theory; i.e., union, intersection, complement, difference

- Uses Venn diagrams to answer questions about sets

Objective 2: Understands calculus concepts and applies knowledge to solve calculus problems

The beginning Mathematics teacher:

A. Understands the meaning of a limit of a function and how to calculate limits of functions, how to determine when the limit does not exist, and how to solve problems using the properties of limits

- Graphically analyzes the limit of $f(x)$ as x approaches a fixed value from both left and right

- Solves limit problems (e.g., a constant times a function, the sum of two functions, the product and quotient of two functions) using properties of limits, where all limits of the individual functions exist at the value that x is approaching

- Analyzes one-sided limits for various functions to see whether or not the limit exists

- Recognizes limits that do not exist, such as $\lim\limits_{x \to 0} \sin\left(\dfrac{1}{x}\right)$ and $\lim\limits_{x \to 0} \dfrac{1}{\sqrt[3]{x^2}}$

B. Understands the derivative of a function as a limit, as the slope of a line tangent to a curve, and as a rate of change

- Constructs a function graph for a given function and a given point $(a, f(a))$, and explains what happens to the succession of slopes of secant lines connecting $(a, f(a))$ to $(x, f(x))$ as x approaches a, from both the right side and the left side

- Uses the limit definition of the derivative to find the derivative of a given function at a given value of x and to find the derivative function

C. Understands how to show that a particular function is continuous

- Applies the three steps (i.e, $f(a)$ exists, $\lim\limits_{x\to a} f(x)$ exists, and $f(a) = \lim\limits_{x\to a} f(x)$) that are part of the definition of what it means for a function to be continuous at $x = a$ to verify whether a given function is continuous at a given point

D. Knows the relationship between continuity and differentiability

- Gives examples of functions that are continuous at $x = a$ but not differentiable at $x = a$, and explains why

E. Understands how and when to use standard differentiation and integration techniques

- Uses standard differentiation techniques

- Uses standard integration techniques

- Understands the relationship between position, velocity, and acceleration functions of a particle in motion

F. Understands how to analyze the behavior of a function; e.g., extrema, concavity, symmetry

- Uses the first and second derivatives to analyze the graph of a function

G. Understands how to apply derivatives to solve problems; e.g., related rates, optimization

- Applies derivatives to solve problems

H. Understands the foundational theorems of calculus; e.g., fundamental theorems of calculus, mean value theorem, intermediate value theorem

- Solves problems using the foundational theorems of calculus

- Understands the relationship between differentiation and integration, including the role of the fundamental theorems of calculus

- Matches graphs of functions with graphs of their derivatives or accumulations

- Understands how to use differentiation and integration of a function to express rates of change and total change

- Understands and calculates the average value of a function over an interval; i.e., mean value theorem of integrals

I. Understands how to use integration to compute area, volume, distance, or other accumulation processes

- Uses integration techniques to compute area, volume, distance, or other accumulation processes

J. Knows how to determine the limits of sequences, if they exist

- Determines the limits of sequences when they exist

K. Is familiar with simple infinite series

- Determines if simple infinite series converge or diverge
- Finds the sum of a simple infinite series if it exists
- Finds the partial sum of a simple infinite series
- Models phenomena (e.g., compound interest, annuities, growth, decay) using finite and infinite arithmetic and geometric sequences and series

Test II Subareas

Subarea	Approx. Percentage of Test
I. Functions	40%
II. Geometry	30%
III. Probability and Statistics	30%

Test II Objectives

Subarea I: Functions

Objective 1: Understands how to interpret functions and apply knowledge to build functions

The beginning Mathematics teacher:

A. Understands the function concept and the use of function notation

- Understands that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range
- Uses function notation, evaluates functions, and interprets statements that use function notation in terms of a context
- Recognizes that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers
- Determines the domain and range of a function from a function rule (e.g., $f(x) = 2x + 1$), graph, set of ordered pairs, or table

B. Understands how function behavior is analyzed using different representations; e.g., graphs, mappings, tables

- For a function that models a relationship between two quantities, interprets key features of graphs and tables (e.g., increasing/decreasing, maximum/minimum, periodicity) in terms of the quantities
- Given a verbal description of a relation, sketches graphs that show key features of that relation
- Graphs functions (i.e., radical, piecewise, absolute value, polynomial, rational, logarithmic, trigonometric) expressed symbolically, and identifies key features of the graph
- Writes a function that is defined by an expression in different but equivalent forms to reveal different properties of the function; e.g., zeros, extreme values, symmetry of the graph

- Interprets the behavior of exponential functions; e.g., growth, decay
- Understands how to determine if a function is odd, even, or neither, and any resulting symmetries

C. Understands how functions and relations are used to model relationships between quantities
- Writes a function that relates two quantities
- Determines an explicit expression or a recursive process that builds a function from a context

D. Understands how new functions are obtained from existing functions; e.g., compositions, transformations, inverses
- Describes how the graph of $g(x)$ is related to the graph of $f(x)$, where $g(x) = f(x) + k$, $g(x) = k\,f(x)$, $g(x) = f(kx)$, or $g(x) = f(x + k)$ for specific values of k (both positive and negative), and finds the value of k given the graphs
- Determines if a function has an inverse and writes an expression for the inverse
- Verifies by composition if one function is the inverse of another
- Given that a function f has an inverse, finds values of the inverse function from a graph or a table of f
- Given a noninvertible function, determines the largest possible domain of the function that produces an invertible function
- Understands the inverse relationship between exponential and logarithmic functions, and uses this relationship to solve problems
- Combines standard function types using arithmetic operations
- Performs domain analysis on functions resulting from arithmetic operations
- Composes functions algebraically, numerically, and graphically
- Performs domain analysis on functions resulting from compositions

Objective 2: Understands and applies knowledge of linear, quadratic, and exponential models and trigonometric functions

The beginning Mathematics teacher:

A. Understands differences between linear, quadratic, and exponential models, including how their equations are created and used to solve problems
- Understands that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals
- Recognizes situations in which one quantity changes at a constant rate per unit interval relative to another

- Recognizes situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another

- Constructs linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two ordered pairs (including reading these from a table)

- Observes that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function

- Expresses the solution to an exponential equation with base b as a logarithm; e.g., $3 \cdot 2^{5t} = 20$, $3 \cdot e^{5t} = 20$

- Uses technology to evaluate logarithms that have any base

- Interprets the parameters in a linear or exponential function in terms of a context; e.g., $A(t) = Pe^{rt}$

- Uses quantities that are inversely related to model phenomena

B. Understands how to construct the unit circle and how to use it to find values of trigonometric functions for all angle measures in their domains

- Finds the values of trigonometric functions of any angle

- Uses the unit circle to explain symmetry and periodicity of trigonometric functions

C. Understands how periodic phenomena are modeled using trigonometric functions

- Chooses trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline

- Understands how to restrict the domain of a trigonometric function so that its inverse can be constructed

- Uses inverse functions to solve trigonometric equations that arise in modeling contexts, and interprets them in terms of the context

D. Understands the application of trigonometric identities (e.g., Pythagorean, double angle, half angle, sum of angles, difference of angles)

- Proves Pythagorean identities (e.g., $\sin^2 \theta + \cos^2 \theta = 1$) and uses them to solve problems

- Uses trigonometric identities to rewrite expressions and solve equations

- Understands trigonometric identities in the context of equivalent graphs of trigonometric functions; e.g., $y = \sin x$ and $y = \cos\left(\dfrac{\pi}{2} - x\right)$ are equivalent graphs

E. Knows how to interpret representations of functions of two variables; e.g., three-dimensional graphs, tables

- Interprets representations of functions of two variables; e.g., $z = f(x, y)$

F. Understands how to solve trigonometric, logarithmic, and exponential equations

- Solves trigonometric, logarithmic, and exponential equations

Subarea II: Geometry

Objective 1: Understands congruence/similarity/triangles/trigonometric ratios and equations for geometric properties

The beginning Mathematics teacher:

A. Understands transformations in a plane

- Knows precise definitions of angle, circle, line segment, perpendicular lines, and parallel lines

- Represents transformations in the plane

- Recognizes whether a transformation preserves distance and angle measure

- Given a rectangle, parallelogram, trapezoid, or regular polygon, describes the rotations and reflections that map it onto itself

- Develops definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments

- Given a geometric figure and a rotation, reflection, or translation, draws the transformed figure

- Specifies a sequence of transformations that will map a given figure onto another figure

B. Understands how to prove geometric theorems such as those about lines and angles, triangles, and parallelograms

- Proves theorems about lines and angles

- Proves theorems about triangles

- Proves theorems about parallelograms

C. Understands how geometric constructions are made with a variety of tools and methods

- Recognizes formal geometric constructions

- Explains how formal geometric constructions are made; e.g., an equilateral triangle, a square, a regular hexagon inscribed in a circle

D. Understands congruence and similarity in terms of transformations

- Uses geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure

- Verifies the properties of dilations given by a center and a scale factor

- Given two figures, uses the definition of congruence in terms of rigid motions to decide if they are congruent

- Given two figures, uses the definition of similarity in terms of dilations to decide if they are similar

- Explains how the criteria for triangle congruence (e.g., ASA, SAS, SSS, HL) follow from the definition of congruence in terms of rigid motions

- Uses the properties of similarity transformations to establish the AA criterion for two triangles to be similar

- Uses congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures

E. Understands how trigonometric ratios are defined in right triangles

- Understands that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles

- Explains and uses the relationship between the sine and cosine of complementary angles

- Uses trigonometric ratios and the Pythagorean theorem to solve right triangles in applied problems

F. Understands how trigonometry is applied to general triangles

- Uses the formula $A = \frac{1}{2}ab\sin(C)$ for the area of a triangle to solve problems

- Applies the Law of Sines and the Law of Cosines to find unknown measurements in triangles

G. Knows how to translate between a geometric description (e.g., focus, asymptotes, directrix) and an equation for a conic section

- Determines and uses the equation of a circle of given center and radius

- Finds the center and radius of a circle given by an equation in standard form

- Determines the equation of a parabola given a focus and directrix

- Determines and uses the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from a point on the curve to the foci is constant

H. Understands how to use coordinate geometry to algebraically prove simple geometric theorems

- Uses coordinates to prove simple geometric theorems algebraically

- Proves the slope criteria for parallel and perpendicular lines, and uses parallel and perpendicular lines to solve geometric problems

- Finds the point on a directed line segment between two given points that partitions the segment in a given ratio

- Uses coordinates to compute perimeters of polygons and areas of triangles and quadrilaterals

Objective 2: Understands circles, geometric measurement and dimension, and modeling with geometry

The beginning Mathematics teacher:

A. Understands and applies theorems about circles

- Identifies and describes relationships among inscribed angles, radii, and chords

- Proves properties of angles for a quadrilateral inscribed in a circle

- Constructs a tangent line from a point outside a given circle to the circle

B. Understands arc length and area measurements of sectors of circles

- Uses the length of the arc intercepted by a central angle or inscribed angle to solve circumference problems

- Uses the formula for the area of a sector to solve problems

C. Understands how perimeter, area, surface area, and volume formulas are used to solve problems

- Uses the perimeter and area of geometric shapes to solve problems

- Uses the surface area and volume of prisms, cylinders, pyramids, cones, and spheres to solve problems

D. Knows how to visualize relationships (e.g., cross section, nets, rotations) between two-dimensional and three-dimensional objects

- Identifies the shapes of two-dimensional cross sections of three-dimensional objects, and identifies three-dimensional objects generated by rotations of two-dimensional objects

- Uses two-dimensional representations of three-dimensional objects to visualize and solve problems

E. Knows how to apply geometric concepts in real-world situations

- Uses geometric shapes, their measures, and their properties to describe objects

- Applies geometric methods to solve design problems

F. Understands the properties of parallel and perpendicular lines, triangles, quadrilaterals, polygons, and circles and how they can be used in problem solving

- Solves problems involving parallel, perpendicular, and intersecting lines

- Applies angle relationships (e.g., supplementary, vertical, alternate interior) to solve problems

- Solves problems that involve medians, midpoints, and altitudes

- Solves problems involving special triangles; e.g., isosceles, equilateral, right
- Knows geometric properties of and relationships among quadrilaterals; e.g., parallelograms, trapezoids
- Solves problems involving angles and diagonals
- Solves problems involving polygons with more than four sides

Subarea III: Probability and Statistics

Objective 1: Understands how to interpret categorical and quantitative data, make inferences, and justify conclusions

The beginning Mathematics teacher:

A. Understands how to summarize, represent, and interpret data collected from measurements on a single variable; e.g., box plots, dot plots, normal distributions

- Represents data with plots on the real number line; e.g., dot plots, histograms, and box plots
- Uses statistics appropriate to the shape of the data distribution to compare center (e.g., median, mean) and spread (e.g., interquartile range, standard deviation) of two or more different data sets
- Interprets differences in shape, center, and spread in the context of the data sets, accounting for possible effects of outliers
- Uses the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages, and recognizes that there are data sets for which such a procedure is not appropriate

B. Understands how to summarize, represent, and interpret data collected from measurements on two variables, either categorical or quantitative; e.g., scatterplots, time series

- Summarizes and interprets categorical data for two categories in two-way frequency tables; e.g., joint, marginal, conditional relative frequencies
- Recognizes possible associations and trends in the data
- Represents data for two quantitative variables on a scatterplot, and describes how the variables are related

C. Understands how to create and interpret linear regression models; e.g., rate of change, intercepts, correlation coefficient

- Uses technology to fit a function to data (i.e., linear regression) and determines a linear correlation coefficient
- Uses functions fitted to data to solve problems in the context of the data
- Assesses the fit of a function by plotting and analyzing residuals

- Interprets the slope and the intercept of a regression line in the context of the data

- Interprets a linear correlation coefficient

- Distinguishes between correlation and causation

D. Understands statistical processes and how to evaluate them

- Understands statistics as a process for making inferences about population parameters based on a random sample from that population

- Decides if a specified model is consistent with results from a given data-generating process; e.g., using simulation

E. Understands how to make inferences and justify conclusions from samples, experiments, and observational studies

- Recognizes the purposes of and differences among sample surveys, experiments, and observational studies, and explains how randomization relates to each

- Uses data from a sample survey to estimate a population mean or proportion

- Uses data from a randomized experiment to compare two treatments

- Uses results of simulations to decide if differences between parameters are significant

- Evaluates reports based on data

Objective 2: Understands conditional probability, the rules of probability, and using probability to make decisions

The beginning Mathematics teacher:

A. Understands the concepts of independence and conditional probability and how to apply these concepts to data

- Describes events as subsets of a sample space using characteristics of the outcomes, or as unions, intersections, or complements of other events

- Understands that two events, *A* and *B*, are independent if and only if $P(A \cap B) = P(A) \cdot P(B)$

- Understands the conditional probability of *A* given *B* as $\dfrac{P(A \text{ and } B)}{P(B)}$, and interprets independence of *A* and *B* as saying that $P(A|B) = P(A)$ and $P(B|A) = P(B)$

B. Understands how to compute probabilities of simple events, probabilities of compound events, and conditional probabilities

- Calculates probabilities of simple and compound events

- Constructs and interprets two-way frequency tables of data when two categories are associated with each object being classified; uses the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities

- Finds $P(A|B)$, and interprets it in terms of a given model

- Applies the addition rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interprets it in terms of a given model

- Applies the general multiplication rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$, and interprets it in terms of a given model

- Calculates probabilities using the binomial probability distribution

C. Knows how to make informed decisions using probabilities and expected values

- Defines a random variable for a quantity of interest by assigning a numerical value to each event in a sample space, and graphs the corresponding probability distribution using the same graphical displays as for data distributions

- Calculates the expected value of a random variable, and interprets it as the mean of the probability distribution

- Develops a probability distribution for a random variable, defined for a sample space in which theoretical probabilities can be calculated, and finds the expected value

- Develops a probability distribution for a random variable, defined for a sample space in which probabilities are assigned empirically, and finds the expected value

- Weighs the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values

- Analyzes decisions and strategies using probability concepts; e.g., fairness

D. Understands how to use simulations to construct experimental probability distributions and to make informal inferences about theoretical probability distributions

- Given the results of simulations, constructs experimental probability distributions

- Given the results of simulations, makes informal inferences about theoretical probability distributions

E. Understands how to find probabilities involving finite sample spaces and independent trials

- Uses the fundamental counting principle to find probabilities involving finite sample spaces and independent trials