GACE® Mathematics Assessment Test II (023)
Curriculum Crosswalk

	Required Coursework Numbers														
Subarea I. Functions (40%)															
Objective 1: Understands how to interpret functions and apply knowledge to build functions															
A. Understands the function concept and the use of function notation															
• Understands that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range															
• Uses function notation, evaluates functions, and interprets statements that use function notation in terms of a context															
• Recognizes that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers															
• Determines the domain and range of a function from a function rule (e.g., $f(x) = 2x + 1$), graph, set of ordered pairs, or table															
B. Understands how function behavior is analyzed using different representations; e.g., graphs, mappings, tables															
• For a function that models a relationship between two quantities, interprets key features of graphs and tables (e.g., increasing/decreasing, maximum/minimum, periodicity) in terms of the quantities															

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• Given a verbal description of a relation, sketches graphs that show key features of that relation															
• Graphs functions (i.e., radical, piecewise, absolute value, polynomial, rational, logarithmic, trigonometric) expressed symbolically, and identifies key features of the graph															
• Writes a function that is defined by an expression in different but equivalent forms to reveal different properties of the function; e.g., zeros, extreme values, symmetry of the graph															
• Interprets the behavior of exponential functions; e.g., growth, decay															
• Understands how to determine if a function is odd, even, or neither, and any resulting symmetries															
C. Understands how functions and relations are used to model relationships between quantities															
• Writes a function that relates two quantities															
• Determines an explicit expression or a recursive process that builds a function from a context															
D. Understands how new functions are obtained from existing functions; e.g., compositions, transformations, inverses															
• Describes how the graph of $g(x)$ is related to the graph of $f(x)$, where $g(x) = f(x) + k$, $g(x) = k\,f(x)$, $g(x) = f(kx)$, or $g(x) = f(x + k)$ for specific values of k (both positive and negative), and finds the value of k given the graphs															

	Required Coursework Numbers														
• Determines if a function has an inverse and writes an expression for the inverse															
• Verifies by composition if one function is the inverse of another															
• Given that a function *f* has an inverse, finds values of the inverse function from a graph or a table of *f*															
• Given a noninvertible function, determines the largest possible domain of the function that produces an invertible function															
• Understands the inverse relationship between exponential and logarithmic functions, and uses this relationship to solve problems															
• Combines standard function types using arithmetic operations															
• Performs domain analysis on functions resulting from arithmetic operations															
• Composes functions algebraically, numerically, and graphically															
• Performs domain analysis on functions resulting from compositions															
Objective 2: Understands and applies knowledge of linear, quadratic, and exponential models and trigonometric functions															
A. Understands differences between linear, quadratic, and exponential models, including how their equations are created and used to solve problems															

	Required Coursework Numbers														
• Understands that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals															
• Recognizes situations in which one quantity changes at a constant rate per unit interval relative to another															
• Recognizes situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another															
• Constructs linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two ordered pairs (including reading these from a table)															
• Observes that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function															
• Expresses the solution to an exponential equation with base b as a logarithm; e.g., $3 \bullet 2^{5t} = 20$, $3 \bullet e^{5t} = 20$															
• Uses technology to evaluate logarithms that have any base															
• Interprets the parameters in a linear or exponential function in terms of a context; e.g., $A(t) = Pe^{rt}$															
• Uses quantities that are inversely related to model phenomena															

	Required Coursework Numbers														
B. Understands how to construct the unit circle and how to use it to find values of trigonometric functions for all angle measures in their domains															
• Finds the values of trigonometric functions of any angle															
• Uses the unit circle to explain symmetry and periodicity of trigonometric functions															
C. Understands how periodic phenomena are modeled using trigonometric functions															
• Chooses trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline															
• Understands how to restrict the domain of a trigonometric function so that its inverse can be constructed															
• Uses inverse functions to solve trigonometric equations that arise in modeling contexts, and interprets them in terms of the context															
D. Understands the application of trigonometric identities (e.g., Pythagorean, double angle, half angle, sum of angles, difference of angles)															
• Proves Pythagorean identities (e.g., $\sin^2 \theta + \cos^2 \theta = 1$) and uses them to solve problems															
• Uses trigonometric identities to rewrite expressions and solve equations															

	Required Coursework Numbers														
• Understands trigonometric identities in the context of equivalent graphs of trigonometric functions; e.g., $y = \sin x$ and $y = \cos\left(\frac{\pi}{2} - x\right)$ are equivalent graphs															
E. Knows how to interpret representations of functions of two variables; e.g., three-dimensional graphs, tables															
• Interprets representations of functions of two variables; e.g., $z = f(x, y)$															
F. Understands how to solve trigonometric, logarithmic, and exponential equations															
• Solves trigonometric, logarithmic, and exponential equations															
Subarea II. Geometry (30%)															
Objective 1: Understands congruence/ similarity/triangles/trigonometric ratios and equations for geometric properties															
A. Understands transformations in a plane															
• Knows precise definitions of angle, circle, line segment, perpendicular lines, and parallel lines															
• Represents transformations in the plane															
• Recognizes whether a transformation preserves distance and angle measure															
• Given a rectangle, parallelogram, trapezoid, or regular polygon, describes the rotations and reflections that map it onto itself															

	Required Coursework Numbers														
• Develops definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments															
• Given a geometric figure and a rotation, reflection, or translation, draws the transformed figure															
• Specifies a sequence of transformations that will map a given figure onto another figure															
B. Understands how to prove geometric theorems such as those about lines and angles, triangles, and parallelograms															
• Proves theorems about lines and angles															
• Proves theorems about triangles															
• Proves theorems about parallelograms															
C. Understands how geometric constructions are made with a variety of tools and methods															
• Recognizes formal geometric constructions															
• Explains how formal geometric constructions are made; e.g., an equilateral triangle, a square, a regular hexagon inscribed in a circle															
D. Understands congruence and similarity in terms of transformations															
• Uses geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure															

	Required Coursework Numbers														
• Verifies the properties of dilations given by a center and a scale factor															
• Given two figures, uses the definition of congruence in terms of rigid motions to decide if they are congruent															
• Given two figures, uses the definition of similarity in terms of dilations to decide if they are similar															
• Explains how the criteria for triangle congruence (e.g., ASA, SAS, SSS, HL) follow from the definition of congruence in terms of rigid motions															
• Uses the properties of similarity transformations to establish the AA criterion for two triangles to be similar															
• Uses congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures															
E. Understands how trigonometric ratios are defined in right triangles															
• Understands that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles															
• Explains and uses the relationship between the sine and cosine of complementary angles															
• Uses trigonometric ratios and the Pythagorean theorem to solve right triangles in applied problems															

	Required Coursework Numbers														
F. Understands how trigonometry is applied to general triangles															
• Uses the formula $A = \frac{1}{2}ab\sin(C)$ for the area of a triangle to solve problems															
• Applies the Law of Sines and the Law of Cosines to find unknown measurements in triangles															
G. Knows how to translate between a geometric description (e.g., focus, asymptotes, directrix) and an equation for a conic section															
• Determines and uses the equation of a circle of given center and radius															
• Finds the center and radius of a circle given by an equation in standard form															
• Determines the equation of a parabola given a focus and directrix															
• Determines and uses the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from a point on the curve to the foci is constant															
H. Understands how to use coordinate geometry to algebraically prove simple geometric theorems															
• Uses coordinates to prove simple geometric theorems algebraically															
• Proves the slope criteria for parallel and perpendicular lines, and uses parallel and perpendicular lines to solve geometric problems															

	Required Coursework Numbers														
• Finds the point on a directed line segment between two given points that partitions the segment in a given ratio															
• Uses coordinates to compute perimeters of polygons and areas of triangles and quadrilaterals															
Objective 2: Understands circles, geometric measurement and dimension, and modeling with geometry															
A. Understands and applies theorems about circles															
• Identifies and describes relationships among inscribed angles, radii, and chords															
• Proves properties of angles for a quadrilateral inscribed in a circle															
• Constructs a tangent line from a point outside a given circle to the circle															
B. Understands arc length and area measurements of sectors of circles															
• Uses the length of the arc intercepted by a central angle or inscribed angle to solve circumference problems															
• Uses the formula for the area of a sector to solve problems															
C. Understands how perimeter, area, surface area, and volume formulas are used to solve problems															
• Uses the perimeter and area of geometric shapes to solve problems															
• Uses the surface area and volume of prisms, cylinders, pyramids, cones, and spheres to solve problems															

	Required Coursework Numbers														
D. Knows how to visualize relationships (e.g., cross section, nets, rotations) between two-dimensional and three-dimensional objects															
• Identifies the shapes of two-dimensional cross sections of three-dimensional objects, and identifies three-dimensional objects generated by rotations of two-dimensional objects															
• Uses two-dimensional representations of three-dimensional objects to visualize and solve problems															
E. Knows how to apply geometric concepts in real-world situations															
• Uses geometric shapes, their measures, and their properties to describe objects															
• Applies geometric methods to solve design problems															
F. Understands the properties of parallel and perpendicular lines, triangles, quadrilaterals, polygons, and circles and how they can be used in problem solving															
• Solves problems involving parallel, perpendicular, and intersecting lines															
• Applies angle relationships (e.g., supplementary, vertical, alternate interior) to solve problems															

	Required Coursework Numbers														
• Solves problems that involve medians, midpoints, and altitudes															
• Solves problems involving special triangles; e.g., isosceles, equilateral, right															
• Knows geometric properties of and relationships among quadrilaterals; e.g., parallelograms, trapezoids															
• Solves problems involving angles and diagonals															
• Solves problems involving polygons with more than four sides															
Subarea III. Probability and Statistics (30%)															
Objective 1: Understands how to interpret categorical and quantitative data, make inferences, and justify conclusions															
A. Understands how to summarize, represent, and interpret data collected from measurements on a single variable; e.g., box plots, dot plots, normal distributions															
• Represents data with plots on the real number line; e.g., dot plots, histograms, and box plots															
• Uses statistics appropriate to the shape of the data distribution to compare center (e.g., median, mean) and spread (e.g., interquartile range, standard deviation) of two or more different data sets															
• Interprets differences in shape, center, and spread in the context of the data sets, accounting for possible effects of outliers															

	Required Coursework Numbers														
• Uses the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages, and recognizes that there are data sets for which such a procedure is not appropriate															
B. Understands how to summarize, represent, and interpret data collected from measurements on two variables, either categorical or quantitative; e.g., scatterplots, time series															
• Summarizes and interprets categorical data for two categories in two-way frequency tables; e.g., joint, marginal, conditional relative frequencies															
• Recognizes possible associations and trends in the data															
• Represents data for two quantitative variables on a scatterplot, and describes how the variables are related															
C. Understands how to create and interpret linear regression models; e.g., rate of change, intercepts, correlation coefficient															
• Uses technology to fit a function to data (i.e., linear regression) and determines a linear correlation coefficient															
• Uses functions fitted to data to solve problems in the context of the data															
• Assesses the fit of a function by plotting and analyzing residuals															
• Interprets the slope and the intercept of a regression line in the context of the data															
• Interprets a linear correlation coefficient															

	Required Coursework Numbers														
• Distinguishes between correlation and causation															
D. Understands statistical processes and how to evaluate them															
• Understands statistics as a process for making inferences about population parameters based on a random sample from that population															
• Decides if a specified model is consistent with results from a given data-generating process; e.g., using simulation															
E. Understands how to make inferences and justify conclusions from samples, experiments, and observational studies															
• Recognizes the purposes of and differences among sample surveys, experiments, and observational studies, and explains how randomization relates to each															
• Uses data from a sample survey to estimate a population mean or proportion															
• Uses data from a randomized experiment to compare two treatments															
• Uses results of simulations to decide if differences between parameters are significant															
• Evaluates reports based on data															
Objective 2: Understands conditional probability, the rules of probability, and using probability to make decisions															

	Required Coursework Numbers														
A. Understands the concepts of independence and conditional probability and how to apply these concepts to data															
• Describes events as subsets of a sample space using characteristics of the outcomes, or as unions, intersections, or complements of other events															
• Understands that two events, *A* and *B*, are independent if and only if $P(A \cap B) = P(A) \cdot P(B)$															
• Understands the conditional probability of *A* given *B* as $\frac{P(A \text{ and } B)}{P(B)}$, and interprets independence of *A* and *B* as saying that $P(A \mid B) = P(A)$ and															
B. Understands how to compute probabilities of simple events, probabilities of compound events, and conditional probabilities															
• Calculates probabilities of simple and compound events															
• Constructs and interprets two-way frequency tables of data when two categories are associated with each object being classified; uses the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities															
• Finds $P(A \mid B)$, and interprets it in terms of a given model															

	Required Coursework Numbers														
• Applies the addition rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interprets it in terms of a given model															
• Applies the general multiplication rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B\|A) = P(B)P(A\|B)$, and interprets it in terms of a given model															
• Calculates probabilities using the binomial probability distribution															
C. Knows how to make informed decisions using probabilities and expected values															
• Defines a random variable for a quantity of interest by assigning a numerical value to each event in a sample space, and graphs the corresponding probability distribution using the same graphical displays as for data distributions															
• Calculates the expected value of a random variable, and interprets it as the mean of the probability distribution															
• Develops a probability distribution for a random variable, defined for a sample space in which theoretical probabilities can be calculated, and finds the expected value															
• Develops a probability distribution for a random variable, defined for a sample space in which probabilities are assigned empirically, and finds the expected value															
• Weighs the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values															

	Required Coursework Numbers														
• Analyzes decisions and strategies using probability concepts; e.g., fairness															
D. Understands how to use simulations to construct experimental probability distributions and to make informal inferences about theoretical probability distributions															
• Given the results of simulations, constructs experimental probability distributions															
• Given the results of simulations, makes informal inferences about theoretical probability distributions															
E. Understands how to find probabilities involving finite sample spaces and independent trials															
• Uses the fundamental counting principle to find probabilities involving finite sample spaces and independent trials															