

NOTATIONS

(a, b)	$\{x : a < x < b\}$
$[a, b)$	$\{x : a \leq x < b\}$
$(a, b]$	$\{x : a < x \leq b\}$
$[a, b]$	$\{x : a \leq x \leq b\}$
$\gcd(m, n)$	<u>greatest common divisor</u> of two integers m and n
$\text{lcm}(m, n)$	<u>least common multiple</u> of two integers m and n
$[x]$	<u>greatest integer</u> m such that $m \leq x$
$m \equiv k \pmod{n}$	m and k are <u>congruent modulo n</u> (m and k have the same remainder when divided by n , or equivalently, $m - k$ is a multiple of n)
f^{-1}	<u>inverse</u> of an invertible function f (<u>not</u> the same as $\frac{1}{f}$)
$\lim_{x \rightarrow a^+} f(x)$	<u>right-hand limit</u> of $f(x)$; limit of $f(x)$ as x approaches a from the right
$\lim_{x \rightarrow a^-} f(x)$	<u>left-hand limit</u> of $f(x)$; limit of $f(x)$ as x approaches a from the left
\emptyset	the empty set
$x \in S$	x is an element of set S
$S \subset T$	set S is a proper subset of set T
$S \subseteq T$	either set S is a proper subset of set T or $S = T$
$S \cup T$	union of sets S and T
$S \cap T$	intersection of sets S and T

DEFINITIONS

A relation \mathfrak{R} on a set S is

reflexive if $x \mathfrak{R} x$ for all $x \in S$

symmetric if $x \mathfrak{R} y \Rightarrow y \mathfrak{R} x$ for all $x, y \in S$

transitive if $(x \mathfrak{R} y \text{ and } y \mathfrak{R} z) \Rightarrow x \mathfrak{R} z$ for all $x, y, z \in S$

antisymmetric if $(x \mathfrak{R} y \text{ and } y \mathfrak{R} x) \Rightarrow x = y$ for all $x, y \in S$

An equivalence relation is a reflexive, symmetric, and transitive relation.

FORMULAS

Sum

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

Half-angle (sign depends on the quadrant of $\frac{\theta}{2}$)

$$\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}}$$

$$\cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}}$$

Range of Inverse Trigonometric Functions

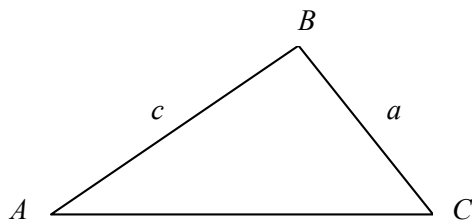
$$\sin^{-1} x \quad \left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$$

$$\cos^{-1} x \quad [0, \pi]$$

$$\tan^{-1} x \quad \left(-\frac{\pi}{2}, \frac{\pi}{2} \right)$$

Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



Law of Cosines

$$c^2 = a^2 + b^2 - 2ab(\cos C)$$

DeMoivre's Theorem

$$(\cos \theta + i \sin \theta)^k = \cos(k\theta) + i \sin(k\theta)$$

Coordinate Transformation

Rectangular (x, y) to polar (r, θ) : $r^2 = x^2 + y^2$; $\tan \theta = \frac{y}{x}$ if $x \neq 0$

Polar (r, θ) to rectangular (x, y) : $x = r \cos \theta$; $y = r \sin \theta$

Distance from point (x_1, y_1) to line $Ax + By + C = 0$

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

Volume

Sphere with radius r : $V = \frac{4}{3}\pi r^3$

Right circular cone with height h and base of radius r : $V = \frac{1}{3}\pi r^2 h$

Right circular cylinder with height h and base of radius r : $V = \pi r^2 h$

Pyramid with height h and base of area B : $V = \frac{1}{3}Bh$

Right prism with height h and base of area B : $V = Bh$

Surface Area

Sphere with radius r : $A = 4\pi r^2$

Right circular cone with radius r and slant height s : $A = \pi r s + \pi r^2$

Differentiation

$$(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$$

$$(f(g(x)))' = f'(g(x))g'(x)$$

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2} \text{ if } g(x) \neq 0$$

Integration by Parts

$$\int u dv = uv - \int v du$$